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Transfer of temporal coherence in parametric down-conversion

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We show that in parametric down-conversion the coherence properties of a temporally partially coherent pump field get entirely transferred to the down-converted entangled two-photon field. Under the assumption that the frequency bandwidth of the down-converted signal-idler photons is much larger than that of the pump, we derive the temporal coherence functions for the down-converted field, for both infinitely fast and time-averaged detection schemes. We show that in each scheme the coherence function factorizes into two separate coherence functions with one of them carrying the entire statistical information of the pump field. In situations in which the pump is a Gaussian Schell-model field, we derive explicit expressions for the coherence functions. Finally, we show that the concurrence of time-energy-entangled two-qubit states is bounded by the degree of temporal coherence of the pump field. This study can have important implications for understanding how correlations of the pump field manifest as two-particle entanglement as well as for harnessing energy-time entanglement for long-distance quantum communication protocols.

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1. INTRODUCTION

Coherence and entanglement are intimately related concepts. The recent attempts at developing a resource-based theory of coherence also reveal such relations [1–4]. One of the physical processes in which the relations between coherence and entanglement can be systematically explored is parametric down-conversion (PDC)—a nonlinear optical process in which a pump photon interacts with a nonlinear crystal to produce a pair of entangled photons, termed as signal and idler [5]. Using the PDC photons, coherence and entanglement effects have been observed in several degrees of freedom including polarization [6], time-energy [7–16], position and momentum [17–19], and orbital angular momentum (OAM) [20–23].

There have been several studies on how coherence and entanglement properties of the down-converted field are affected by different PDC settings and pump field parameters 29–24 ,17]]. However, regarding how the intrinsic correlations of the pump field get transferred to manifest as two-photon coherence and entanglement, there have been efforts mostly in the polarization and spatial degrees of freedom [19,26,30,31]. In the spatial degree of freedom, a very general spatially partially coherent field was considered and it was shown that the spatial coherence properties of the pump field get entirely transferred to that of the down-converted two-photon field [19]. However, in the temporal degree of freedom, the effects due to the temporal correlations of the pump field have only been studied in two limiting situations: one, in which the constituent frequency components are completely correlated (fully coherent pulsed field) [15,32–36] and the other, in which the constituent frequency components are completely uncorrelated (continuous-wave field) [7–14,37–43]. In this paper, we study the coherence transfer in PDC for a general temporally partially coherent pump field, and explicitly quantify this correlation transfer for the special case of a partially coherent Gaussian Schell-model field [44] in which the correlations between the constituent frequency components have a Gaussian distribution.

The paper is organized as follows. In Section 2, we consider a general temporally partially coherent pump field and show that its temporal coherence properties get entirely transferred to the down-converted two-photon field. We work out the two-photon temporal coherence functions for both infinitely fast and time-averaged detection schemes and show that in each scheme the coherence function factorizes into two separate coherence functions with one of them carrying the entire statistical information of the pump field. In Section 4, we show that the entanglement of time-energy-entangled two-qubit states is bounded by the degree of temporal coherence of the partially coherent pump field. We present our conclusions in Section 5.

2. TRANSFER OF TEMPORAL COHERENCE IN PDC

A. Detection with Infinitely Fast Detectors

We follow the formalism worked out in Ref. [11] and represent a general two-alternative two-photon interference of the PDC photons by the two-photon path diagrams shown in Fig. 1. The pump is a general temporally partially coherent field. Alternatives 1 and 2 are the two pathways by which a pump photon is down-converted, and the down-converted signal and idler photons are detected in coincidence at single-photon detectors D_s and D_i, respectively. There are six independent time parameters in this setting. The subscripts p, s, and i denote the pump, signal, and idler, respectively. We adopt the convention that a signal photon is the one that arrives at detector D_s, while an idler photon is the one that arrives at detector D_i. The symbol τ denotes the traversal time of a photon, while ϕ denotes the phase, other than the dynamical phase, accumulated by a photon. Thus, τ_{s1} denotes the traversal time of the signal photon in alternative 1, etc. The various signal, idler, and pump quantities are used to define the following parameters:

$$\Delta\tau \equiv \tau_1 - \tau_2 \equiv \left(\tau_{p1} + \frac{\tau_{s1} + \tau_{i1}}{2}\right) - \left(\tau_{p2} + \frac{\tau_{s2} + \tau_{i2}}{2}\right),$$

$$\Delta\tau' \equiv \tau_1' - \tau_2' \equiv \left(\frac{\tau_{s1} - \tau_{i1}}{2}\right) - \left(\frac{\tau_{s2} - \tau_{i2}}{2}\right),$$

$$\Delta\phi \equiv \phi_1 - \phi_2 \equiv (\phi_{p1} + \phi_{s1} + \phi_{i1}) - (\phi_{p2} + \phi_{s2} + \phi_{i2}). \quad \textbf{(1)}$$

The parameters defined above are identical to those defined in Ref. [11], except for τ_1', τ_2', and $\Delta\tau'$, which have been scaled down by a factor of 2. It is found that this rescaling imparts the equations in this paper a neat and symmetric form. The two-photon state $|\psi\rangle_1$ produced in alternative 1 in the weak down-conversion limit is given by [32,37,45]

$$|\psi\rangle_1 = A_1 \iint \mathrm{d}\omega_s \mathrm{d}\omega_i V(\omega_p)\Phi_1(\omega_s, \omega_i) \times e^{i(\omega_p\tau_{p1} + \phi_{p1})}|\omega_s\rangle_{\omega_{s1}}|\omega_i\rangle_{\omega_{i1}}, \quad \textbf{(2)}$$

where $V(\omega_p)$ is the random spectral amplitude of the pump field at frequency ω_p, and $\Phi_1(\omega_s, \omega_i)$ is the phase-matching

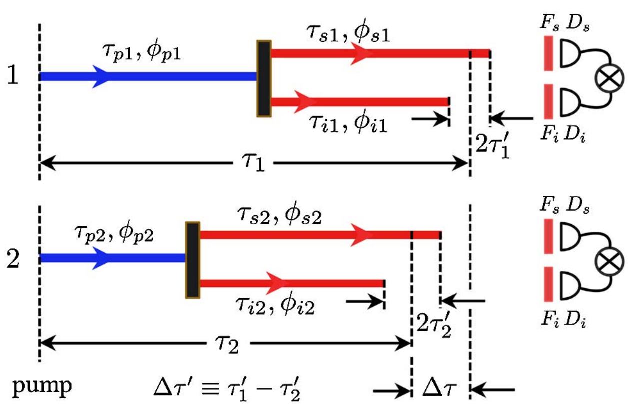

Fig. 1. Schematic representation of two-photon interference using the two-photon path diagrams. Alternatives 1 and 2 are the two pathways by which a pump photon is down-converted and the down-converted photons are detected at single-photon detectors D_s and D_i.

function in alternative 1. The two-photon state $|\psi\rangle_2$ in alternative 2 can be similarly defined. The complete two-photon state $|\psi\rangle$ at the detectors is the sum of the two-photon states in alternatives 1 and 2 and can be written as $|\psi\rangle = |\psi\rangle_1 + |\psi\rangle_2$. The corresponding density matrix ρ of the state at the detectors is therefore

$$\hat{\rho} = \langle|\psi\rangle\langle\psi|\rangle. \quad \textbf{(3)}$$

Here $\langle\cdots\rangle$ represents an ensemble average over infinitely many realizations of the two-photon state.

We now denote the positive frequency parts of the electric fields at detectors D_s and D_i by $\hat{E}_s^{(+)}(t)$ and $\hat{E}_i^{(+)}(t)$, respectively, and write them as

$$\hat{E}_s^{(+)}(t) = \kappa_{s1}\hat{E}_{s1}^{(+)}(t - \tau_{s1}) + \kappa_{s2}\hat{E}_{s2}^{(+)}(t - \tau_{s2}), \quad \textbf{(4a)}$$

$$\hat{E}_i^{(+)}(t) = \kappa_{i1}\hat{E}_{i1}^{(+)}(t - \tau_{i1}) + \kappa_{i2}\hat{E}_{i2}^{(+)}(t - \tau_{i2}), \quad \textbf{(4b)}$$

where $\kappa_{s1(2)}$ and $\kappa_{i1(2)}$ are scalar amplitudes and where

$$\hat{E}_{s1}^{(+)}(t - \tau_{s1}) = e^{i\phi_{s1}} \int_0^\infty \mathrm{d}\omega f_{s1}(\omega - \omega_{s0})\hat{a}_{s1}(\omega)e^{-i\omega(t-\tau_{s1})} \quad \textbf{(5)}$$

is the positive frequency part of the electric field at detector D_s in alternative 1, etc. The function $f_{s1}(\omega - \omega_{s0})$ is the amplitude transmission function of the filter F_s placed at detector D_s, etc. The filters F_s and F_i are centered at frequencies ω_{s0} and ω_{i0}, respectively, and we assume the phase-matching condition $\omega_{p0} = \omega_{s0} + \omega_{i0}$, where ω_{p0} is the central frequency of the pump field $V(\omega_p)$. The coincidence count rate $R_{si}^{(2)}(t_s, t_i)$ of the two detectors is the probability per (unit time)^2 that a signal photon is detected at time t_s and the corresponding idler photon is detected at time t_i, and it is given by $R_{si}^{(2)}(t_s, t_i) = \mathrm{Tr}\{\hat{\rho}\hat{E}_s^{(-)}(t_s)\hat{E}_i^{(-)}(t_i)\hat{E}_i^{(+)}(t_i)\hat{E}_s^{(+)}(t_s)\}$ [46]. Using the definitions and expressions of Eqs. (1)–(5), we evaluate $R_{si}^{(2)}(t_s, t_i)$ to be

$$R_{si}^{(2)}(t_s, t_i) = \kappa_1^2 R^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}) + \kappa_2^2 R^{(2)}(t_s, t_i, \tau_{s2}, \tau_{i2}) + \kappa_1\kappa_2\Gamma^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}, \tau_{s2}, \tau_{i2})e^{-i\Delta\phi} + \mathrm{c.c.}, \quad \textbf{(6a)}$$

where $\kappa_1 = \kappa_{s1}\kappa_{i1}$, $\kappa_2 = \kappa_{s2}\kappa_{i2}$,

$$\Gamma^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}, \tau_{s2}, \tau_{i2}) = \mathrm{Tr}\{\hat{\rho}\hat{E}_{s1}^{(-)}(t_s - \tau_{s1}) \times \hat{E}_{i1}^{(-)}(t_i - \tau_{i1})\hat{E}_{i2}^{(+)}(t_i - \tau_{i2})\hat{E}_{s2}^{(+)}(t_s - \tau_{s2})\}, \quad \textbf{(6b)}$$

and

$$R^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}) = \Gamma^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}, \tau_{s1}, \tau_{i1}). \quad \textbf{(6c)}$$

Equation (6) is the interference law for the two-photon field. The first and the second terms are the coincidence count rates in alternatives 1 and 2, respectively. The interference term $\Gamma^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}, \tau_{s2}, \tau_{i2})$ appears when both the alternatives are present, and it will be referred to as the two-photon cross-correlation function of the down-converted field. We now make the assumption that the spectral width $\Delta\omega_{p0}$ of the pump field is much smaller than the central frequency ω_{p0} and the spectral widths of the phase-matching functions and filter functions. As a result, the phase-matching and filter functions can be taken to be approximately constant in the frequency range $(\omega_{p0} - \Delta\omega_{p0}/2, \omega_{p0} + \Delta\omega_{p0}/2)$. This assumption remains valid for most PDC experiments employing continuous-wave pump fields [7–14,47] and pulsed pump fields [15,32–34,36] and may only be invalid for experiments employing ultrashort pulsed pump fields [48–50]. We use the relations $\omega_p = \omega_s + \omega_i$, $\omega_d = \omega_s - \omega_i$, $\omega_{p0} = \omega_{s0} + \omega_{i0}$, $\omega_{d0} = \omega_{s0} - \omega_{i0}$ and define the integration variables $\bar{\omega}_p = \omega_p - \omega_{p0}$ and $\bar{\omega}_d = \omega_d - \omega_{d0}$. Using Eqs. (1)–(6), we obtain after a long but straightforward calculation

$$\Gamma^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}, \tau_{s2}, \tau_{i2}) = \Gamma_p\left(\tau_1 - \frac{t_s + t_i}{2}, \tau_2 - \frac{t_s + t_i}{2}\right) \times \Gamma_d\left(\tau_1' - \frac{t_s - t_i}{2}, \tau_2' - \frac{t_s - t_i}{2}\right), \quad (7)$$

where

$$\Gamma_p\left(\tau_1 - \frac{t_s + t_i}{2}, \tau_2 - \frac{t_s + t_i}{2}\right) = e^{-i\omega_{p0}\Delta\tau} \iint d\bar{\omega}_p' d\bar{\omega}_p'' \langle V^*(\bar{\omega}_p' + \omega_{p0}) V(\bar{\omega}_p'' + \omega_{p0})\rangle \times \exp\left[-i\bar{\omega}_p'\left(\tau_1 - \frac{t_s + t_i}{2}\right)\right] \exp\left[i\bar{\omega}_p''\left(\tau_2 - \frac{t_s + t_i}{2}\right)\right], \quad (8)$$

and

$$\Gamma_d\left(\tau_1' - \frac{t_s - t_i}{2}, \tau_2' - \frac{t_s - t_i}{2}\right) = e^{-i\omega_{d0}\Delta\tau'} \iint d\bar{\omega}_d' d\bar{\omega}_d'' \langle g_1^*(\bar{\omega}_d') g_2(\bar{\omega}_d'')\rangle \times \exp\left[-i\bar{\omega}_d'\left(\tau_1' - \frac{t_s - t_i}{2}\right)\right] \exp\left[i\bar{\omega}_d''\left(\tau_2' - \frac{t_s - t_i}{2}\right)\right], \quad (9)$$

with

$$g_1(\omega) = \Phi_1\left(\omega_{s0} + \frac{\omega}{2}, \omega_{i0} - \frac{\omega}{2}\right) f_{s1}\left(\frac{\omega}{2}\right) f_{i1}\left(-\frac{\omega}{2}\right),$$

etc. The ensemble average $\langle V^*(\bar{\omega}_p' + \omega_{p0}) V(\bar{\omega}_p'' + \omega_{p0})\rangle$ is the cross-spectral density function of the pump field. It is at once clear from Eq. (8) that the coherence function $\Gamma_p(\tau_1 - \frac{t_s+t_i}{2}, \tau_2 - \frac{t_s+t_i}{2})$ and the cross-spectral density function $\langle V^*(\bar{\omega}_p' + \omega_{p0}) V(\bar{\omega}_p'' + \omega_{p0})\rangle$ are connected through the generalized Wiener–Khintchine relation [51] with parameters $\tau_1 - (t_s + t_i)/2$ and $\tau_2 - (t_s + t_i)/2$. So, in terms of the two-photon time parameters $\tau_1 - (t_s + t_i)/2$ and $\tau_2 - (t_s + t_i)/2$, the coherence function $\Gamma_p(\tau_1 - \frac{t_s+t_i}{2}, \tau_2 - \frac{t_s+t_i}{2})$ has the same functional form as that of the cross-correlation function of the pump field. The function $\langle g_1^*(\bar{\omega}_d') g_2(\bar{\omega}_d'')\rangle$ is also in the form of a cross-spectral density function, and as is clear from Eq. (9), it forms a generalized Wiener–Khintchine relation with the coherence function $\Gamma_d(\tau_1' - \frac{t_s-t_i}{2}, \tau_2' - \frac{t_s-t_i}{2})$. Therefore, the function $\Gamma_d(\tau_1' - \frac{t_s-t_i}{2}, \tau_2' - \frac{t_s-t_i}{2})$ not only carries all the information about the phase-matching conditions and the crystal parameters but also carries information about any statistical randomness that the down-converted photons go through [52]. It is interesting to note that any statistical randomness encountered by the photons after the down-conversion affects only $\Gamma_d(\tau_1' - \frac{t_s-t_i}{2}, \tau_2' - \frac{t_s-t_i}{2})$ and has no effect on $\Gamma_p(\tau_1 - \frac{t_s+t_i}{2}, \tau_2 - \frac{t_s+t_i}{2})$. This fact can potentially be used for encoding information in the pump's coherence function and decoding if from the down-converted photons even after the down-converted photons have passed through turbulent media.

We thus find that the two-photon cross-correlation function factorizes into two separate coherence functions. The coherence function $\Gamma_p(\tau_1 - \frac{t_s+t_i}{2}, \tau_2 - \frac{t_s+t_i}{2})$ carries the entire statistical information of the pump field, and in this way the temporal correlation properties of the pump photon get entirely transferred to the down-converted photons. This result is the temporal analog of the effect described in Ref. [19], in which it was shown that in PDC the spatial coherence properties of the pump field gets entirely transferred to the down-converted two-photon field. However, the present paper extends beyond just establishing this analogy. For example, in Ref. [19], the effect due to the phase-matching function was completely ignored, but in the present paper, we have included it through the coherence function $\Gamma_d(\tau_1' - \frac{t_s-t_i}{2}, \tau_2' - \frac{t_s-t_i}{2})$. Moreover, like most spatial-interference schemes, Ref. [19] does not employ a detection scheme that involves spaceaveraging. However, most time-domain experiments employ time-averaged detection schemes. Therefore, in the present paper, we also work out how time-averaged detection schemes affect the temporal coherence transfer in PDC.

B. Time-Averaged Detection Scheme

In most experiments, one does not measure the instantaneous coincidence rate $R_{si}^{(2)}(t_s, t_i)$ of Eq. (6). Instead, one measures the time-averaged coincidence count rate, averaged over the photon collection time $T_{\rm pc}$ and the coincidence time-window $T_{\rm ci}$. The time-averaged two-photon cross-correlation function $\bar{\Gamma}^{(2)}$ can be found by first expressing it as

$$\bar{\Gamma}^{(2)} = \langle\langle\Gamma^{(2)}(t_s, t_i, \tau_{s1}, \tau_{i1}, \tau_{s2}, \tau_{i2})\rangle\rangle_{t_s, t_i} = \left\langle \Gamma_p\left(\tau_1 - \frac{t_s + t_i}{2}, \tau_2 - \frac{t_s + t_i}{2}\right)\right\rangle_{\frac{t_s+t_i}{2}} \times \left\langle \Gamma_d\left(\tau_1' - \frac{t_s - t_i}{2}, \tau_2' - \frac{t_s - t_i}{2}\right)\right\rangle_{\frac{t_s-t_i}{2}}, \quad (10)$$

and then integrating it with respect to $(t_s + t_i)/2$ over $T_{\rm pc}$, and with respect to $(t_s - t_i)/2$ over $T_{\rm ci}$. In most experiments, the coincidence time-window $T_{\rm ci}$ spans a few nanoseconds, which is much longer than the inverse frequency bandwidth of $g(\omega)$, typically of the order of picoseconds. The photon collection time $T_{\rm pc}$ is usually a few seconds and is much longer than the inverse frequency bandwidth of the pump field $V(\omega_p)$, typically of the order of microseconds. Thus we perform the above time-averaging in the limit $T_{pc}, T_{ci} \to \infty$ to obtain

$$\bar{\Gamma}^{(2)} = \bar{\Gamma}_p(\tau_1, \tau_2)\bar{\Gamma}_d(\tau_1', \tau_2') = \sqrt{\bar{I}_1 \bar{I}_2}\sqrt{\bar{G}_1 \bar{G}_2}\bar{\gamma}_p(\Delta\tau)\bar{\gamma}_d(\Delta\tau') = \bar{R}^{(2)}\bar{\gamma}_p(\Delta\tau)\bar{\gamma}_d(\Delta\tau'). \quad (11)$$

Here $\bar{I}_1 = \bar{\Gamma}_p(\tau_1, \tau_1)$, $\bar{G}_1 = \bar{\Gamma}_d(\tau_1', \tau_1')$, $\bar{R}^{(2)} \equiv \sqrt{\bar{I}_1 \bar{I}_2}\sqrt{\bar{G}_1 \bar{G}_2}$, $\bar{\gamma}_p(\Delta\tau) = \bar{\Gamma}_p(\tau_1, \tau_2)/\sqrt{\bar{I}_1 \bar{I}_2}$, and $\bar{\gamma}_d(\Delta\tau) = \bar{\Gamma}_d(\tau_1', \tau_2')/\sqrt{\bar{G}_1 \bar{G}_2}$, etc. The function $\bar{\gamma}_p(\Delta\tau)$ satisfies $0 \le |\bar{\gamma}_p(\Delta\tau)| \le 1$ and diminishes over a $\Delta\tau$-scale given by the inverse pump bandwidth $1/\Delta\omega_{p0}$. The function $\bar{\gamma}_d(\Delta\tau')$ also satisfies

$0 \le |\bar{\gamma}_d(\Delta\tau')| \le 1$ and diminishes over a $\Delta\tau'$-scale given by the inverse frequency bandwidth $1/\Delta\omega_{d0}$. The temporal widths of $\bar{\gamma}_p(\Delta\tau)$ and $\bar{\gamma}_d(\Delta\tau')$ limit the ranges over which fringes could be observed as functions of $\Delta\tau'$ and $\Delta\tau$, respectively, in a time-averaged two-photon interference experiment.

The coincidence count rate of Eq. (6) in the time-averaged scheme therefore becomes

$$\bar{R}_{si}^{(2)} = \kappa_1^2 \bar{R}^{(2)} + \kappa_2^2 \bar{R}^{(2)} + \kappa_1\kappa_2 \bar{R}^{(2)} \bar{\gamma}_p(\Delta\tau)\bar{\gamma}_d(\Delta\tau') \times e^{i(\omega_{p0}\Delta\tau + \omega_{d0}\Delta\tau' + \Delta\phi)} + \text{c.c.} \quad (12)$$

A similar expression was reported in Ref. [11], where various temporal two-photon interference effects have been described. The time-averaged coherence function $\bar{\gamma}_p(\Delta\tau)$ has the same functional form as the time-averaged coherence function of the pump field. The time-averaged coherence function $\bar{\gamma}_d(\Delta\tau')$ depends on the phase-matching function and the crystal parameters, and its functional form shows up in the Hong–Ou–Mandel (HOM) [7] and HOM-like effects [10,53].

3. SPECIAL CASE OF A GAUSSIAN SCHELL-MODEL PUMP FIELD

In the last section, we considered PDC with a very general nonstationary pump field and described how the temporal coherence properties of the pump field get transferred to the down-converted two-photon field. In this section, we consider the pump field to be a widely studied class of nonstationary fields, namely, the Gaussian Schell-model field, also known as the nonstationary Gaussian pulsed fields [44].

The cross-spectral density function of a Gaussian Schell-model field is given by [44]

$$\langle V^*(\omega'' + \omega_0) V(\omega' + \omega_0)\rangle = A \exp\left[-\frac{(\omega'^2 + \omega''^2)}{4(\Delta\omega_{p0})^2}\right] \exp\left[-\frac{(\omega' - \omega'')^2}{2(\Delta\omega_c)^2}\right], \quad (13)$$

where $\Delta\omega_{p0}$ is the frequency bandwidth of the field. The parameter $\Delta\omega_c$ is called the spectral correlation width and it quantifies the frequency separation up to which different frequency components are phase-correlated. The limit $\Delta\omega_c \to 0$ corresponds to a continuous-wave, stationary field, in which case the constituent frequency components are completely uncorrelated. The other limit $\Delta\omega_c \to \infty$ corresponds to a fully coherent pulsed field, in which case the constituent frequency components are perfectly phase-correlated. The corresponding temporal correlation function $\Gamma_{GS}(t_1, t_2)$ can be calculated by using the generalized Wiener–Khintchine theorem [44],

$$\Gamma_{GS}(t_1, t_2) = \sqrt{I(t_1)I(t_2)}\gamma_{GS}(\Delta t), \quad (14)$$

with $\Delta t = t_1 - t_2$, and where

$$I(t_{1(2)}) = \frac{2\pi\Delta\omega_{p0}A}{T} \exp\left[-\frac{t_{1(2)}^2}{2T^2}\right],$$

$$\text{and} \quad \gamma_{GS}(\Delta t) = \exp\left[-\frac{(\Delta t)^2}{2\tau_{coh}^2}\right].$$

Here $\tau_{coh} = (\Delta\omega_c/\Delta\omega_{p0})[1/(2\Delta\omega_{p0})^2 + 1/(\Delta\omega_c)^2]^{1/2}$ is a measure of the coherence time of the field and $T = [1/(2\Delta\omega_{p0})^2 + 1/(\Delta\omega_c)^2]^{1/2}$ is a measure of the temporal width of the nonstationary Gaussian pulse. The limit $\Delta\omega_c \to \infty$ yields $\tau_{coh} \to \infty$, as expected for a fully coherent field, and the other limit $\Delta\omega_c \to 0$ yields $\tau_{coh} = 1/\Delta\omega_{p0}$, as expected for a continuous-wave, stationary field.

Now, for conceptual clarity, we assume in this section that $\Gamma_d(\tau_1' - \frac{t_s - t_i}{2}, \tau_2' - \frac{t_s - t_i}{2}) = 1$ and take the pump field to be the Gaussian Schell-model field given by Eq. (13). Equation (7) then becomes

$$\Gamma^{(2)}(t_s, t_i, \tau_1, \tau_2) = \Gamma_p\left(\tau_1 - \frac{t_s + t_i}{2}, \tau_2 - \frac{t_s + t_i}{2}\right) = \sqrt{I\left(\tau_1 - \frac{t_s + t_i}{2}\right) I\left(\tau_2 - \frac{t_s + t_i}{2}\right)}\gamma_p(\Delta\tau), \quad (15)$$

where

$$I\left(\tau_1 - \frac{t_s + t_i}{2}\right) = \frac{2\pi\Delta\omega_{p0}A}{T} \exp\left[-\frac{(\tau_1 - \frac{t_s + t_i}{2})^2}{2T^2}\right], \text{etc.,}$$

$$\text{and} \quad \gamma_p(\Delta\tau) = \exp\left[-\frac{\Delta\tau^2}{2\tau_{coh}^2}\right].$$

As expected from Eq. (7), we find that in terms of $\tau_1 - \frac{t_s + t_i}{2}$ and $\tau_2 - \frac{t_s + t_i}{2}$, the two-photon cross-correlation function in Eq. (15) assumes the same functional form as does the cross-correlation function in Eq. (14) in terms of t_1 and t_2. When integrated over t, Eq. (15) yields

$$\bar{\Gamma}^{(2)} = \bar{\Gamma}_p(\tau_1, \tau_2) = \sqrt{\bar{I}_1\bar{I}_2}\bar{\gamma}_p(\Delta\tau), \quad (16)$$

with $\bar{I}_1 = \bar{I}_2 = (2\pi)^{\frac{3}{2}}\Delta\omega_{p0}A$ and

$$\bar{\gamma}_p(\Delta\tau) = \exp\left[-\frac{\Delta\tau^2}{2\bar{\tau}_{coh}^2}\right],$$

where $\bar{\tau}_{coh} = 1/\Delta\omega_{p0}$ is a measure of the coherence time. The time averaging washes out effects due to frequency correlations. Thus, only in the case of a stationary pump field $\bar{\gamma}_p(\Delta\tau) = \gamma_p(\Delta\tau)$ and $\bar{\tau}_{coh} = \tau_{coh}$.

4. PUMP TEMPORAL COHERENCE AND TWO-QUBIT ENERGY-TIME ENTANGLEMENT

Two-qubit states are the necessary ingredients for many quantum information protocols [54–56] and have been realized by exploiting the entanglement of PDC photons in several degrees of freedom including polarization [57], time-energy [12,15,16,34,40,47,58], position-momentum [18,59,60], and orbital angular momentum (OAM) [21,61–63]. There have been previous studies describing how correlations of the pump field in polarization and spatial degrees of freedom affect the entanglement of the generated two-qubit states. In the polarization degree of freedom, it was shown [31] that the degree of polarization P of the pump photon puts an upper bound of $(1 + P)/2$ on the concurrence of the generated two-qubit state. In the spatial degree of freedom, effects of pump spatial coherence on the entanglement of the generated spatial two-qubit state have been worked out for two-qubit states that have only two nonzero diagonal elements, and for such states it has been shown that the concurrence is bounded by the degree of spatial coherence of the pump field [19]. However, to the best of our knowledge, no such relation has so far been derived for the time-energy entangled two-qubit states.

There are two generic methods by which one makes a PDC-based time-energy entangled two-qubit state. In the first method, one uses a continuous-wave pump field, either single-mode [12] or multimode [40,47]. In the second method, one uses a pulsed pump field [15,16,34,58]. In both these methods, a combination of post-selection strategies, such as selecting a faster coincidence detection window, using arrival time of pump photon as a trigger, etc., one makes sure that there are only two alternative pathways in which the signal and idler photons reach their respective detectors. The two alternative pathways form the two-dimensional qubit space for the signal and idler photons. We represent by $|s1\rangle$ the state of the signal photon in alternative 1, etc. Therefore, the density matrix $\rho_{2\text{qubit}}$ of the two-qubit state can be written in the basis $\{|s1\rangle|i1\rangle, |s1\rangle|i2\rangle, |s2\rangle|i1\rangle, |s2\rangle|i2\rangle\}$ as

$$\rho = \begin{pmatrix} a & 0 & 0 & c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ c^* & 0 & 0 & b \end{pmatrix}, \quad \textbf{(17)}$$

where the diagonal terms a and b are the probabilities that the signal and idler photons are detected in states $|s1\rangle|i1\rangle$ and $|s2\rangle|i2\rangle$, respectively, and the off-diagonal term c is a measure of coherence between states $|s1\rangle|i1\rangle$ and $|s2\rangle|i2\rangle$. In an experimental situation, the density matrix ρ can be represented by the two alternative pathways of Fig. 1. Therefore, using Eq. (12), we write $a = \eta\kappa_1^2\bar{R}^{(2)}$ and $b = \eta\kappa_2^2\bar{R}^{(2)}$, where $\eta = 1/[\kappa_1^2\bar{R}^{(2)} + \kappa_2^2\bar{R}^{(2)}]$ is the constant of proportionality. The off-diagonal term is given by

$$c = \eta\kappa_1\kappa_2\bar{R}^{(2)}\bar{\gamma}_p(\Delta\tau)\bar{\gamma}_d(\Delta\tau')e^{i(\omega_{p0}\Delta\tau+\omega_{d0}\Delta\tau'+\Delta\phi)}. \quad \textbf{(18)}$$

The entanglement of $\rho_{2\,\text{qubit}}$, as quantified by Wootters's concurrence $C(\rho_{2\,\text{qubit}})$ [64], can be shown to be

$$C(\rho_{2\,\text{qubit}}) = 2|c| = \frac{2\kappa_1\kappa_2\bar{R}^{(2)}}{\kappa_1^2\bar{R}^{(2)} + \kappa_2^2\bar{R}^{(2)}}\bar{\gamma}_p(\Delta\tau)\bar{\gamma}_d(\Delta\tau'). \quad \textbf{(19)}$$

The pre-factor $2\kappa_1\kappa_2\bar{R}^{(2)}/(\kappa_1^2\bar{R}^{(2)} + \kappa_2^2\bar{R}^{(2)})$ is no greater than 1, and $\bar{\gamma}_d(\Delta\tau')$ also satisfies $0 \le |\bar{\gamma}_d(\Delta\tau')| \le 1$. We therefore arrive at the relation $C(\rho_{2\,\text{qubit}}) \le \bar{\gamma}_p(\Delta\tau)$. Therefore, we find that the concurrence $C(\rho_{2\,\text{qubit}})$ of the time-energy two-qubit state is bounded from above by the degree of coherence of the pump photon, and thus that the temporal correlations of the pump field set an upper bound on the attainable concurrence for a two-qubit state of the form of Eq. (17). We note that in situations in which $\bar{\gamma}_d(\Delta\tau') \approx 1$ and $\kappa_1 = \kappa_2$, the maximum achievable concurrence for a pulsed field can be unity in principle, and for a continuous-wave field it can be unity as long as $\Delta\tau$ is much smaller than the coherence time of the pump field. The above result is the temporal analog of the results obtained in the polarization [31] and spatial [19] degrees of freedom. However, unlike in the spatial degree of freedom, which does not involve any space-averaged detection scheme, the results derived in this paper show that even for the time-averaged detection schemes, the temporal correlation properties of the pump do directly decide the upper limit on entanglement that a time-energy entangled two-qubit state can achieve.

5. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have shown that in PDC, the coherence properties of a temporally partially coherent pump field get entirely transferred to the down-converted entangled two-photon field. Under the assumption that the frequency bandwidth of the down-converted signal-idler photons is much larger than that of the pump, we have worked out the temporal coherence functions of the down-converted field for both infinitely fast and time-averaged detection schemes. We have shown that in each scheme the coherence function factorizes into two separate coherence functions with one of them carrying the entire statistical information of the pump field. Taking the pump to be a Gaussian Schell-model field, we have derived explicit expressions for the coherence functions. Finally, we have shown that the concurrence of time-energy entangled two-qubit states is bounded by the degree of temporal coherence of the pump field. This result extends previously obtained results in the spatial [19] and polarization [31] degrees of freedom to the temporal degree of freedom and can thus have important implications for understanding how correlations of the pump field in general manifest as two-particle entanglement. Our results can also be important for time-energy two-qubit-based quantum communication applications. This is because it has been recognized that energy-time entangled two-qubit states are better than the polarization two-qubit states for long-distance quantum information [49,50], and our results show that the temporal coherence properties of the pump field can be used as a parameter for tailoring the two-qubit time-energy entanglement. Moreover, it is known that the purity of the individual photon states increases with the decrease in the entanglement of the two-photon state. Therefore, our work can also have implications for PDC-based heralded single-photon sources [65,66] in the sense that the degree of purity of heralded photons can be tailored by controlling the coherence properties of the pump field.

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REFERENCES

1. W. Vogel and J. Sperling, "Unified quantification of nonclassicality and entanglement," Phys. Rev. A **89**, 052302 (2014).
2. T. Baumgratz, M. Cramer, and M. B. Plenio, "Quantifying coherence," Phys. Rev. Lett. **113**, 140401 (2014).
3. D. Girolami, "Observable measure of quantum coherence in finite dimensional systems," Phys. Rev. Lett. **113**, 170401 (2014).
4. E. Chitambar and M.-H. Hsieh, "Relating the resource theories of entanglement and quantum coherence," Phys. Rev. Lett. **117**, 020402 (2016).
5. D. C. Burnham and D. L. Weinberg, "Observation of simultaneity in parametric production of optical photon pairs," Phys. Rev. Lett. **25**, 84–87 (1970).
6. J. Brendel, W. Dultz, and W. Martienssen, "Geometric phases in two-photon interference experiments," Phys. Rev. A **52**, 2551–2556 (1995).

7. C. K. Hong, Z. Y. Ou, and L. Mandel, "Measurement of subpicosecond time intervals between two photons by interference," Phys. Rev. Lett. **59**, 2044–2046 (1987).
8. X. Y. Zou, L. J. Wang, and L. Mandel, "Induced coherence and indistinguishability in optical interference," Phys. Rev. Lett. **67**, 318–321 (1991).
9. T. J. Herzog, J. G. Rarity, H. Weinfurter, and A. Zeilinger, "Frustrated two-photon creation via interference," Phys. Rev. Lett. **72**, 629–632 (1994).
10. T. B. Pittman, D. V. Strekalov, A. Migdall, M. H. Rubin, A. V. Sergienko, and Y. H. Shih, "Can two-photon interference be considered the interference of two photons?" Phys. Rev. Lett. **77**, 1917–1920 (1996).
11. A. K. Jha, M. N. O'sullivan, K. W. C. Chan, and R. W. Boyd, "Temporal coherence and indistinguishability in two-photon interference effects," Phys. Rev. A **77**, 021801(R) (2008).
12. J. D. Franson, "Bell inequality for position and time," Phys. Rev. Lett. **62**, 2205–2208 (1989).
13. J. Brendel, E. Mohler, and W. Martienssen, "Time-resolved dual-beam two-photon interferences with high visibility," Phys. Rev. Lett. **66**, 1142–1145 (1991).
14. A. K. Jha, M. Malik, and R. W. Boyd, "Exploring energy-time entanglement using geometric phase," Phys. Rev. Lett. **101**, 180405 (2008).
15. J. Brendel, N. Gisin, W. Tittel, and H. Zbinden, "Pulsed energy-time entangled twin-photon source for quantum communication," Phys. Rev. Lett. **82**, 2594–2597 (1999).
16. R. T. Thew, S. Tanzilli, W. Tittel, H. Zbinden, and N. Gisin, "Experimental investigation of the robustness of partially entangled qubits over 11 km," Phys. Rev. A **66**, 062304 (2002).
17. E. J. S. Fonseca, C. H. Monken, S. Pádua, and G. A. Barbosa, "Transverse coherence length of down-converted light in the two-photon state," Phys. Rev. A **59**, 1608–1614 (1999).
18. L. Neves, G. Lima, E. J. S. Fonseca, L. Davidovich, and S. Pádua, "Characterizing entanglement in qubits created with spatially correlated twin photons," Phys. Rev. A **76**, 032314 (2007).
19. A. K. Jha and R. W. Boyd, "Spatial two-photon coherence of the entangled field produced by down-conversion using a partially spatially coherent pump beam," Phys. Rev. A **81**, 013828 (2010).
20. E. Nagali, L. Sansoni, F. Sciarrino, F. De Martini, L. Marrucci, B. Piccirillo, E. Karimi, and E. Santamato, "Optimal quantum cloning of orbital angular momentum photon qubits through Hong-Ou-Mandel coalescence," Nat. Photonics **3**, 720–723 (2009).
21. A. K. Jha, J. Leach, B. Jack, S. Franke-Arnold, S. M. Barnett, R. W. Boyd, and M. J. Padgett, "Angular two-photon interference and angular two-qubit states," Phys. Rev. Lett. **104**, 010501 (2010).
22. H. Di Lorenzo Pires, H. C. B. Florijn, and M. P. van Exter, "Measurement of the spiral spectrum of entangled two-photon states," Phys. Rev. Lett. **104**, 020505 (2010).
23. A. K. Jha, G. S. Agarwal, and R. W. Boyd, "Partial angular coherence and the angular schmidt spectrum of entangled two-photon fields," Phys. Rev. A **84**, 063847 (2011).
24. C. K. Hong and L. Mandel, "Theory of parametric frequency down conversion of light," Phys. Rev. A **31**, 2409–2418 (1985).
25. M. H. Rubin, "Transverse correlation in optical spontaneous parametric down-conversion," Phys. Rev. A **54**, 5349–5360 (1996).
26. C. H. Monken, P. H. S. Ribeiro, and S. Pádua, "Transfer of angular spectrum and image formation in spontaneous parametric down-conversion," Phys. Rev. A **57**, 3123–3126 (1998).
27. A. Joobeur, B. E. A. Saleh, T. S. Larchuk, and M. C. Teich, "Coherence properties of entangled light beams generated by parametric down-conversion: theory and experiment," Phys. Rev. A **53**, 4360–4371 (1996).
28. P. H. Souto Ribeiro, "Partial coherence with twin photons," Phys. Rev. A **56**, 4111–4117 (1997).
29. B. E. A. Saleh, M. C. Teich, and A. V. Sergienko, "Wolf equations for two-photon light," Phys. Rev. Lett. **94**, 223601 (2005).
30. S. P. Walborn, A. N. de Oliveira, R. S. Thebaldi, and C. H. Monken, "Entanglement and conservation of orbital angular momentum in spontaneous parametric down-conversion," Phys. Rev. A **69**, 023811 (2004).
31. G. Kulkarni, V. Subrahmanyam, and A. K. Jha, "Intrinsic upper bound on two-qubit polarization entanglement predetermined by pump polarization correlations in parametric down-conversion," Phys. Rev. A **93**, 063842 (2016).
32. W. P. Grice and I. A. Walmsley, "Spectral information and distinguishability in type-II down-conversion with a broadband pump," Phys. Rev. A **56**, 1627–1634 (1997).
33. T. E. Keller and M. H. Rubin, "Theory of two-photon entanglement for spontaneous parametric down-conversion driven by a narrow pump pulse," Phys. Rev. A **56**, 1534–1541 (1997).
34. W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, "Quantum cryptography using entangled photons in energy-time bell states," Phys. Rev. Lett. **84**, 4737–4740 (2000).
35. Y. M. Mikhailova, P. A. Volkov, and M. V. Fedorov, "Biphoton wave packets in parametric down-conversion: spectral and temporal structure and degree of entanglement," Phys. Rev. A **78**, 062327 (2008).
36. T. Inagaki, N. Matsuda, O. Tadanaga, M. Asobe, and H. Takesue, "Entanglement distribution over 300 km of fiber," Opt. Express **21**, 23241–23249 (2013).
37. Z. Y. Ou, L. J. Wang, and L. Mandel, "Vacuum effects on interference in two-photon down conversion," Phys. Rev. A **40**, 1428–1435 (1989).
38. M. H. Rubin, D. N. Klyshko, Y. H. Shih, and A. V. Sergienko, "Theory of two-photon entanglement in type-II optical parametric down-conversion," Phys. Rev. A **50**, 5122–5133 (1994).
39. P. W. Milonni, H. Fearn, and A. Zeilinger, "Theory of two-photon down-conversion in the presence of mirrors," Phys. Rev. A **53**, 4556–4566 (1996).
40. O. Kwon, Y.-S. Ra, and Y.-H. Kim, "Coherence properties of spontaneous parametric down-conversion pumped by a multi-mode cw diode laser," Opt. Express **17**, 13059–13069 (2009).
41. M. B. Nasr, S. Carrasco, B. E. A. Saleh, A. V. Sergienko, M. C. Teich, J. P. Torres, L. Torner, D. S. Hum, and M. M. Fejer, "Ultrabroadband biphotons generated via chirped quasi-phase-matched optical parametric down-conversion," Phys. Rev. Lett. **100**, 183601 (2008).
42. M. Okano, H. H. Lim, R. Okamoto, N. Nishizawa, S. Kurimura, and S. Takeuchi, "0.54 μm resolution two-photon interference with dispersion cancellation for quantum optical coherence tomography," Sci. Rep. **5**, 18042 (2015).
43. A. Tanaka, R. Okamoto, H. H. Lim, S. Subashchandran, M. Okano, L. Zhang, L. Kang, J. Chen, P. Wu, T. Hirohata, S. Kurimura, and S. Takeuchi, "Noncollinear parametric fluorescence by chirped quasi-phase matching for monocycle temporal entanglement," Opt. Express **20**, 25228–25238 (2012).
44. P. Pääkkönen, J. Turunen, P. Vahimaa, A. T. Friberg, and F. Wyrowski, "Partially coherent Gaussian pulses," Opt. Commun. **204**, 53–58 (2002).
45. L. J. Wang, X. Y. Zou, and L. Mandel, "Induced coherence without induced emission," Phys. Rev. A **44**, 4614–4622 (1991).
46. R. J. Glauber, "The quantum theory of optical coherence," Phys. Rev. **130**, 2529–2539 (1963).
47. O. Kwon, K.-K. Park, Y.-S. Ra, Y.-S. Kim, and Y.-H. Kim, "Time-bin entangled photon pairs from spontaneous parametric down-conversion pumped by a cw multi-mode diode laser," Opt. Express **21**, 25492–25500 (2013).
48. I. Marcikic, H. de Riedmatten, W. Tittel, V. Scarani, H. Zbinden, and N. Gisin, "Time-bin entangled qubits for quantum communication created by femtosecond pulses," Phys. Rev. A **66**, 062308 (2002).
49. I. Marcikic, H. De Riedmatten, W. Tittel, H. Zbinden, and N. Gisin, "Long-distance teleportation of qubits at telecommunication wavelengths," Nature **421**, 509–513 (2003).
50. I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, M. Legré, and N. Gisin, "Distribution of time-bin entangled qubits over 50 km of optical fiber," Phys. Rev. Lett. **93**, 180502 (2004).
51. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University, 1995).
52. A. K. Jha, G. A. Tyler, and R. W. Boyd, "Effects of atmospheric turbulence on the entanglement of spatial two-qubit states," Phys. Rev. A **81**, 053832 (2010).

53. D. V. Strekalov, T. B. Pittman, and Y. H. Shih, “What we can learn about single photons in a two-photon interference experiment,” Phys. Rev. A **57**, 567–570 (1998).
54. A. K. Ekert, “Quantum cryptography based on Bell’s theorem,” Phys. Rev. Lett. **67**, 661–663 (1991).
55. C. H. Bennett and S. J. Wiesner, “Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states,” Phys. Rev. Lett. **69**, 2881–2884 (1992).
56. C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, “Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels,” Phys. Rev. Lett. **70**, 1895–1899 (1993).
57. P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, “New high-intensity source of polarization-entangled photon pairs,” Phys. Rev. Lett. **75**, 4337–4341 (1995).
58. S. Ramelow, L. Ratschbacher, A. Fedrizzi, N. Langford, and A. Zeilinger, “Discrete, tunable color entanglement,” Phys. Rev. Lett. **103**, 253601 (2009).
59. L. Neves, G. Lima, J. G. Aguirre Gómez, C. H. Monken, C. Saavedra, and S. Pádua, “Generation of entangled states of qudits using twin photons,” Phys. Rev. Lett. **94**, 100501 (2005).
60. J. G. Rarity and P. R. Tapster, “Experimental violation of Bell’s inequality based on phase and momentum,” Phys. Rev. Lett. **64**, 2495–2498 (1990).
61. A. Vaziri, G. Weihs, and A. Zeilinger, “Experimental two-photon, three-dimensional entanglement for quantum communication,” Phys. Rev. Lett. **89**, 240401 (2002).
62. N. K. Langford, R. B. Dalton, M. D. Harvey, J. L. O’Brien, G. J. Pryde, A. Gilchrist, S. D. Bartlett, and A. G. White, “Measuring entangled qutrits and their use for quantum bit commitment,” Phys. Rev. Lett. **93**, 053601 (2004).
63. J. Leach, B. Jack, J. Romero, M. Ritsch-Marte, R. W. Boyd, A. K. Jha, S. M. Barnett, S. Franke-Arnold, and M. J. Padgett, “Violation of a Bell inequality in two-dimensional orbital angular momentum state-spaces,” Opt. Express **17**, 8287–8293 (2009).
64. W. K. Wootters, “Entanglement of formation of an arbitrary state of two qubits,” Phys. Rev. Lett. **80**, 2245–2248 (1998).
65. J. L. O’Brien, “Optical quantum computing,” Science **318**, 1567–1570 (2007).
66. K. N. Cassemiro, K. Laiho, and C. Silberhorn, “Accessing the purity of a single photon by the width of the Hong-Ou-Mandel interference,” New J. Phys. **12**, 113052 (2010).